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Few-Body Systems

Nucleon-Nucleon Interaction Models and Non-Locality

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Abstract. The effect of non-locality in the NN interaction models is examined. It is shown that this feature can explain differences in predictions made from models evidencing a difference with this respect. This is done for both static and dynamical observables, taking into account that a non-local term can be transformed away by performing a unitary transformation. Some results for the deuteron form factors, the $A(Q^2)$ structure function and the $T_{20}(Q^2)$ tensor polarization are given as an example. A few cases where discrepancies cannot be explained are also considered. They point to differences in the models as for the deuteron asymptotic normalizations, A_S and A_D, which are not affected by the present analysis.

1 Introduction

Apart from a recent work by Doleschall and Borbély[1], non-locality in the NN interaction has not been the object of dedicated studies for a long time. It is not absent however in models where it appears most often as a by-product of some prejudice in their construction. In the Paris model[2], for instance, it was realized that an energy dependence could help in fitting NN scattering data. The transformation of this energy dependence into a p^2/M dependence provides a non-local component. Later on, the Bonn group produced a model, field-theory motivated, taking into account the coupling of the NN channel to $NN\pi$, NΔ, $\Delta\,\Delta, \cdots$ channels[3]. It contains both a spatial and a time-non-locality. Moreover, the improvement consisting in introducing the Dirac spinors to describe 1/2-spin particles also provides non-locality. One could add other examples that have not been concretized in a high accuracy model. Taking into account the substructure of nucleons and mesons in terms of quarks most often leads to a non-locality[4], which is better expressed in configuration space than in momentum one, contrarily to the above sources of non-locality.

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A double question may be raised about this non-locality. Does it help in explaining NN scattering data and how this could be evidenced? On the other hand, taking into account that a non-locality can be transformed away by a unitary transformation (wave by wave), one can wonder whether the different models on the market are independent of each other?

Concerning the first question, some answer is obtained by examining models such as the versions Nij1 (non-local) and Nij2 (local) of the Nijmegen group[5]. They equally fit the scattering phase shifts (χ^2 per datum = 1.03), but in the first case, this is achieved with 41 parameters while 47 are required in the other one. The slightly smaller number in the former case perhaps provides indication that the introduction of some non-locality is beneficial.

The second question is the main object of the present paper. The plan will be as follows. In the second section, we present the different non-local terms which we are interested in. How they can be removed by a unitary transformation at the first order is given. The third section is devoted to a few selected results concerning the deuteron: static properties, form factors, structure function ($A(Q^2)$) and the tensor polarization ($T_{20}(Q^2)$). It involves a comparison of these quantities obtained with different models when the effect of the non-locality is taken into account. Section four contains the conclusion and a discussion. Due to a lack of space, we concentrate here on the essential points. Details and extended results could be found in refs. [6, 7, 8].

2 Transforming Away Non-Local Terms

The interaction of interest here may be written:

$$V = V_S + V_T + \{\frac{\mathbf{p^2}}{M}, \tilde{W}_S\} + \{\frac{\mathbf{p^2}}{M}, \tilde{W}_T\} + [\frac{\mathbf{p^2}}{M}, i\,U]. \tag{1}$$

where the non-local terms take the form of an anticommutator or a commutator. Another term of same order could be considered but those retained here are the only ones appearing in the pion-exchange contribution when this one is expanded up to order $1/M^4$. Due to its long range, it a priori provides larger contributions. Moreover, they are theoretically well identified while shorter range contributions are likely to have some effective character. The last term in the above equation has been studied at length in refs. [6, 7, 9]. Though it has a different origin, it has a strong similarity with a term arising from the difference of pseudo-scalar and pseudo-vector πNN couplings, considered in an earlier work by Friar[10]. As for the anticommutator terms, only rough estimates were made in the past. They are considered more completely here.

In principle, if two models are unitary equivalent, the corresponding Hamiltonians, H and H', should fulfill the following relation:

$$H = \frac{p^2}{M} + V + V_{NL} = e^{-S}\; H'\; e^S = e^{-S}\;(\frac{p^2}{M} + V')\; e^S. \tag{2}$$

At the first order in the interaction, the quantity, S, appearing in the unitary transformation, $\exp(S)$, can be determined by requiring:

$$V_{NL} + [S, \frac{\mathbf{p^2}}{M}] = \Delta V^0 \ , \tag{3}$$

where ΔV^0 has to be local. This equation is fulfilled as follows:

$$\begin{aligned} S &= iU + \frac{i}{2}\Bigg(\boldsymbol{p}\cdot\boldsymbol{r} \ \Big(V_0(r) + S_{12}(\hat{r})\ V_1(r)\Big) \\ &\quad +\Big(\boldsymbol{\sigma_1}\cdot\boldsymbol{p}\ \boldsymbol{\sigma_2}\cdot\boldsymbol{r} + \boldsymbol{\sigma_2}\cdot\boldsymbol{p}\ \boldsymbol{\sigma_1}\cdot\boldsymbol{r} - \frac{2}{3}\ \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\ \boldsymbol{p}\cdot\boldsymbol{r}\Big)\ V_2(r) + h.c.\Bigg), \end{aligned} \tag{4}$$

with
$$\begin{aligned} V_0(r) &= -\frac{1}{r}\int_r^\infty W_S(r')dr', \\ V_2(r) &= -\frac{1}{r}\int_r^\infty dr' \int_{r'}^\infty \frac{W_T(r'')}{r''}dr'', \\ V_1(r) &= -V_2(r) - \int_r^\infty \frac{W_T(r')}{r'}dr'. \end{aligned} \tag{5}$$

It is noticed that the difference of V and V' in Eq. (2) involves two-body but also many-body terms. Similarly, if a one-body current is introduced in one representation, the other one contains two, ... body currents to ensure the unitary equivalence. The role of the two-body part is examined in the following section. Applications involve the interaction models: Nij2[5], Argonne V18[11], Reid93[5] which are local ones, Nij1[5], Nij93[5], Paris[2] which have a linearly p^2/M dependence, and the Bonn-QB, Bonn-CD ones[3].

3 Results for Static and Dynamical Quantities

Concerning static properties, a quantity of interest is the deuteron D-state probability, often referred to characterize different models. The difference between the Paris and Bonn-QB model is 0.78%. As the two models have quite close values for the mixing parameters, ϵ_1, their comparison is largely free of bias with this respect. Taking into account the effect of the non-local term, V_{NL}, explains 0.74% (0.60% and 0.14% for the commutator and anti-commutator parts respectively). Notice that the change in the deuteron D-state probability just reflects the fact that this quantity is not an observable one. Similar results were reached by von Geramb et al.[12], using the inverse scattering problem methods. Another interesting quantity is the ratio $Q_D/(A_S\ A_D)$, which, contrary to the above one, is observable. The difference is 0.23 fm^3 while V_{NL} explains 0.20 fm^3. It is also instructive to look at quantities that only depend on the scalar anticommutator part of V_{NL}. In this order, we compare the squared charge radius for the models Nij1 and Nij2. The models differ by an amount of 0.004 fm^2 while the effect of the non-locality is estimated to be around 0.001 fm^2. The apparent failure to explain the discrepancy in this case actually points to the difference of the models for the asymptotic normalization A_S, which as is well known, governs the size of the charge radius. This factor is not affected by the present analysis of non-local effects.

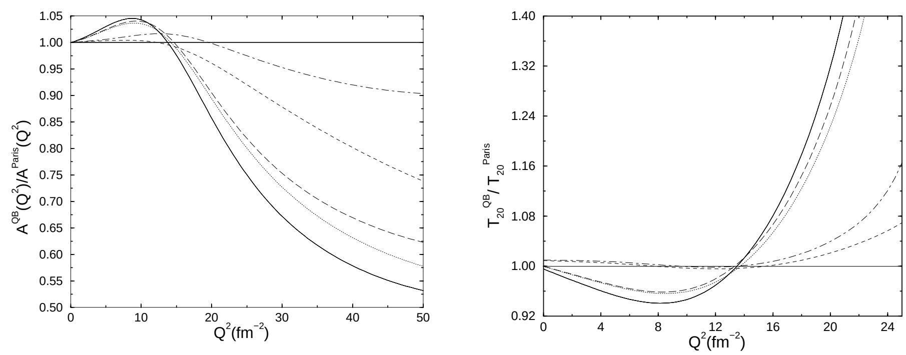

Figure 1. Ratio of predictions for the Bonn-QB and Paris models: effect of non-local terms in the interaction (see text for explanations)

Ratios of form factors and related quantities calculated in different representations of the interaction should go to 1 when the non-locality effect is accounted for and provided that the unitary equivalence is ensured. It is not so in practice because the unitary transformation is treated at first order. In each case, there are thus two sets of results, depending on whether corrections are added to predictions made with one model or removed from the other ones.

Ratios of predictions made from the Bonn-QB and Paris models for the $A(Q^2)$ and $T_{20}(Q^2)$ observables are shown in Fig. 1. As it can be seen, the ratio of bare predictions (continuous line) tends to 1 when the effect of the anticommutator (dashed and dotted lines) and commutator terms (small-dash and dashed-dotted lines) is considered. A large part of the effect is due to the tensor part of V_{NL}. Notice that a slight departure from 1 appears for $T_{20}(Q^2)$ around $Q^2 = 0$. The effect of the scalar part of the anticommutator, which is seen in Fig. 2 (left part), shows features similar to the previous ones. It involves the S-wave function close to the origin, which is generally suppressed in local models compared to non-local ones. The right part of the same figure emphasizes a case where an agreement between two models turns into a disagreement. In fact, this one is consistent with what is expected from the comparison of the asymptotic normalization, unaffected by the present analysis. In Fig. 3, all predictions corrected for the effect of a non-local term are compared to the Paris ones. At low Q^2, it is seen that some discrepancy is still present while the initial motivation of the work was rather to explain it by non-locality effects. Actually, for both $A(Q^2)$ and $T_{20}(Q^2)$, the discrepancy reflects a sensitivity to the A_D/A_S ratio in the first case and A_S in the second one. Around $Q^2 = 20$ fm^{-2}, the ratio becomes very close to 1, while in absence of corrections departures up to 10% and 20% respectively could be observed. The decrease of the uncertainty has motivated Schiavella and Sick in using the quadrupole form factor to derive the neutron charge form factor[13].

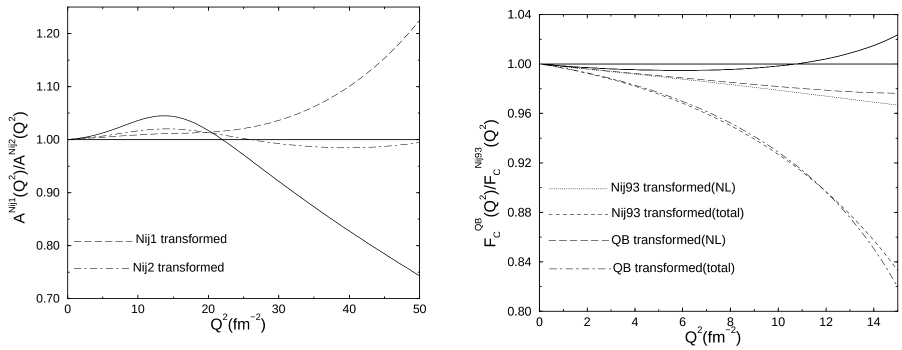

Figure 2. Examples showing a decrease of discrepancies between models due a scalar non-local effect (left part) and the appearance, on the contrary, of a discrepancy (right part); see text for comments

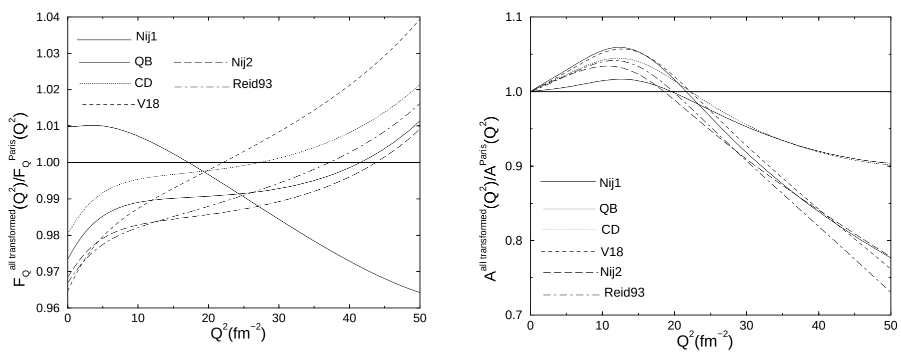

Figure 3. Comparison of predictions of different models after incorporating non-locality corrections (see comments in the text)

4 Conclusion

Effects of non-local terms in NN interaction models have been considered. Roughly, they explain a large part of the differences that the comparison of various model predictions for electromagnetic observables evidences. The part with a tensor character has been found to be the dominant one. It is also the best determined. Similar conclusions hold in some cases for the scalar part but the effect is often masked by other effects related to the fact that models correspond to different values of the asymptotic normalization, A_S, which is an observable and remains unchanged in the analysis performed here.

While the original goal of present studies was rather to relate discrepancies between models to some non-locality, it appears that this is not always possible, especially at low Q^2. Interestingly however, accounting for this effect tends to restore some hierarchy of the results, as expected from simple models. Thus, in the above range, the structure function, $A(Q^2)$, and the quadrupole form factor, $F_Q(Q^2)$, evidence a direct sensitivity respectively to the asymptotic normalizations, A_S and A_D, which are unaffected by the present analysis of non-local effects.

The result of the analysis presented here was not a priori guaranteed. The fact that it points to a unique family of phase-equivalent models indicates that the sensitivty of the models to different parametrizations of the radial part for instance or to a different fit to experimental data is rather small. Thus, the availability of various models is not without interest. The remaining sensitivity, as for A_S or equivalently the scattering length, a_t, strongly calls for a more accurate determination of these quantities.

Throughout the present study, we compare together predictions of models for electromagnetic observables. Ultimately, a comparison to experiment should be done. In this respect, a model to be prefered, most probably non-local, is that one based on degrees of freedom of which effectice character, unavoidable in any case, is as low as possible.

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